MA6459- NUMERICAL METHODS

TWO MARKS QUESTIONS WITH ANSWERS

PART-A

UNIT-I

SOLUTION OF EQUATIONS AND EIGEN VALUE PROBLEMS

1. What is the rate of convergent and convergent condition in Newton Raphson method? (N/D 2015,2014,2012)

Sol:

The rate of convergent in NR Method is of order 2. Convergent condition is $|f(x)f''(x)| < |f'(x)^2|$.

2. Explain the term Pivoting? (A/M 2014,2013)

Sol:

In the elimination process if any one of the pivot element $a_{11}, a_{22}, \ldots\ldots a_{nn}$ vanishes or becomes very small compared in that column then we attempt to rearrange the remaining rows so as to obtain a nonvanishing pivot or to avoid the multiplication by a large number. This strategy is called Pivoting.

3. Define round off error? (A/M 2012)

Sol:

The round off error is the quantity R which must be added to the finite representation of a computed number in order to make it the true representation of that number .

4. Compare Gauss elimination and Gauss Jordan method? (A/M 2012,2011)

Sol:

GAUSS ELIMINATION METHOD	GAUSS JORDON METHOD
1.coefficient matrix transformed in to Upper triangular matrix	coefficient matrix transformed in to diagonal matrix
2.Direct method	Direct method
3. Need back substitution method	No need of back substitution method

5. Write a sufficient condition for Gauss Seidel method to converge? (A/M 2015,2013,2012)

Sol:

The process of iteration by Gauss Seidel method will converge if in each equation of the system the absolute value of the largest coefficient is greater than the sum of the absolute values of the remaining coefficient. The coefficient matrix should be diagonally dominant.

6. Compare the Direct and Indirect (Iterative) Method? (A/M 2013)

Sol:

DIRECT METHOD	INDIRECT METHOD
1.Exact solution	Approximate solution
2.Less time	More time

7. What do you mean by diagonally dominant (A/M 2012,2011)

Sol:

A matrix is diagonally dominant if the numerical value of the leading diagonal element in each row is greater than or equal to the sum of the numerical values of the other values of the other elements in that row.

8.DeriveNewtons algorithm for finding pth root of a number N? (A/M 2015,2014,2013)

Sol:

$x^p - N = 0$

$f(x) = x^p - N = 0$ and $f'(x) = px^{p-1}$

By Newtons algorithm

$$x_{n+1} = x_n - \frac{x_n^p - N}{px_n^{p-1}}$$

$$= \frac{px_n^p - x_n^p + N}{px_n^{p-1}}$$

$$\boxed{x_{n+1} = \frac{(p-1)x_n^p + N}{px_n^{p-1}}}$$

8. What type of eigen value can be obtained using Power method? (A/M 2011)

We can obtain dominant eigen value of the given matrix.

9. Write down the procedure to find the numerically smallest eigen value of a matrix by Power method? A/M 2011)

Sol:

By Power method the largest eigen value of A^{-1} can be found then smallest engen value of A is the reciprocal of largest eigen value of A^{-1}.

10. What are two types of errors involving in Numerical computations? (A/M 2013)

Sol:

Round of error and Truncation error

11. Define Truncation error? (A/M 2011)

Sol:

The error caused by using approximate formula in computation is known as Truncation error.

12. **Write the procedure involved in Gauss Jordan elimination method. (A/M 2015)**

Sol:

The augmented matrix[A,B] of the system AX=B is reduced into [D,K](D is a diagonal matrix)

13. Find the inverse of A=$\begin{pmatrix}1 & 3\\ 2 & 7\end{pmatrix}$by gauss Jordan method.**?** **(N/D 2014)**

Sol:

$$(A/I)\sim\left(\begin{array}{cc|cc}1 & 3 & 1 & 0\\ 0 & 1 & -2 & 1\end{array}\right)$$

$$\sim\left(\begin{array}{cc|cc}1 & 0 & 7 & -3\\ 0 & 1 & -2 & 1\end{array}\right)\sim(I/A^{-1})$$

$$A^{-1}=\begin{pmatrix}7 & -3\\ 2 & 1\end{pmatrix}$$

14. .Using gauss elimination method to solve 5x+4y=15,3x+7y=12. (**M/J 2014)**

Sol:

$$\left(\begin{array}{cc|c}5 & 4 & 15\\ 3 & 7 & 12\end{array}\right)\sim\left(\begin{array}{cc|c}1 & {}^{4}/_{5} & 3\\ 0 & {}^{23}/_{5} & 3\end{array}\right)$$

Y=15/23=0.6522; x+4/5(0.6522)=3

X=2.4783

UNIT-II

INTERPOLATION AND APPROXIMATION

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1. What is inverse Interpolation? **(A/M 2013,2012)**

Sol:

Suppose we are given a table of values of x and y. Direct interpolation is the process of finding the values of y corresponding to the value of x , not present in the table. Inverse interpolation is the process of finding the values of x corresponding to the value of y, not present in the table.

2. What is the assumption we make when Lagrange's formula is used? **(A.M 2011,2012)**

Sol:

Lagrange's interpolation formula can be used whether the values of x, the independent variable are equally spaced (or) not whether the difference of y become smaller (or) not.

3. Using Newton's Backward Difference formula write the formulae for the first and second order derivatives at the end of value $x=x_n$ upto the fourth order difference term. **(M/J 2014, 2012)**

Sol:

$$y(x) = y_n + \frac{n}{1!}\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \frac{n(n+1)(n+2)(n+3)}{4!}\nabla^4 y_n$$

Where n= $\frac{x - x_n}{h}$

Given $x=x_n \rightarrow n=0$

$Y(x) = y_n$, $Y'(x) = y_n' = \lim_{h\to 0}\frac{y(x+h)-y(x)}{h}$, $Y''(x) = y_n''$.

4. State Newton's Backward Difference formula **(N/D 2014,2013)**

Sol:

$$y(x) = y_n + \frac{n}{1!}\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \frac{n(n+1)(n+2)(n+3)}{4!}\nabla^4 y_n$$

Where n= $\frac{x - x_n}{h}$

5. When Newton's backward interpolation formula is used? **(N/D 2011,2014)**

Sol:

To find the unknown values of y for some 'x' which lies at the end of the table of values,we use Newton's backward interpolation formula.

6. Give the Newton's divided difference formula. **(N/D 2012,2013)**

Sol:

$F(x)= f(x_0)+(x-x_0)\, f(x_0, x_1)+ (x-x_0)\,(x-x_1)\, f(x_0, x_1,x_2)+ (x-x_0)\,(x-x_1)\,(x-, x_2)\; f(x_0, x_1,x_2, x_3)+\ldots.$

7. Show that $\Delta^3_{bcd}\left(\frac{1}{a}\right) = \frac{-1}{abcd}$ **(A/M 2015)**

Sol:

If $f(x)=\frac{1}{x},\ f(a)=\frac{1}{a},\ f(a,b)=\underset{b}{\Delta}(\frac{1}{a})=\frac{(\frac{1}{b}-\frac{1}{a})}{b-a}=\frac{-1}{ab}$

$$f(a,b,c)=\frac{f(b,c)-f(a,b)}{c-a}=\frac{(\frac{-1}{bc}+\frac{1}{ab})}{c-a}=\frac{1}{abc}(\frac{c-a}{c-a})=\frac{1}{abc}$$

$$f(a,b,c,d)=\frac{f(b,c,d)-f(a,b,c)}{d-a}=\frac{(\frac{1}{bcd}-\frac{1}{abc})}{d-a}=\frac{-1}{abcd}(\frac{a-d}{a-d})=\frac{-1}{abcd}$$

$$\underset{bcd}{\Delta^3}(\frac{1}{a})=\frac{-1}{abcd}$$

8. Write down the formula for the cubic spline? **(N/D 2014,2011)**

Sol:

$$y(x)=\frac{1}{6}\left[(x_i-x)^3M_{i-1}+(x-x_{i-1})^3M_i\right]+(x_i-x)(y_{i-1}-\frac{1}{6}M_{i-1})+(x-x_{i-1})(y_i-\frac{1}{6}M_i)$$

9. What is cubic spline? **(A/M 2015,M/J 2014)**

Sol:

A cubic polynomial which has continuous slope and curvature is called a cubic spline.

10.Find the divided difference of f(x)=x^3+x+2 for the arguments 1,3. **(N/D 2012,2013)**

Sol:

X	f(x)	$\Delta(f)$
1	**4**	
		$\frac{32-4}{3-1}$ **= 14**
3	**32**	

UNIT-III

NUMERICAL DIFFERENTIATION AND INTEGRATION

1.**State Newton's formula to find f'(x) and f''(x) using forward differences**

Sol: **(A.U 2009,2010,M/D2011)**

Let y=f(x) be a function taking the values $y_0, y_1, \ldots\ldots y_n$ corresponding to $x_0, x_1, \ldots\ldots x_n$ of the independent variable x. Let the values of x be at equidistant intervals of size h.

Then f'(x) $=\frac{dy}{dx}=\frac{1}{h}\left[\Delta y_0+\frac{(2u-1)}{1.2}\,\Delta^2 y_0+\frac{(3u^2-6u+2)}{1.2.3}\Delta^3 y_0+\cdots\right]$

Where u$=\frac{x-x_0}{h}$

In particular,at $x=x_0$,u=0

$\left(\frac{dy}{dx}\right)=\frac{1}{h}\left[\Delta y_0-\frac{\Delta^2}{2}y_0+\frac{\Delta^3}{3}y_0+\cdots\right]$

$\left(\frac{d^2y}{dx^2}\right)=\frac{1}{h^2}\left[\Delta^2 y_0-\Delta^3 y_0+\frac{11}{12}\Delta^4 y_0+\cdots\right]$

2.**Using Newton's backward difference formula,write the formulae for the first and second derivatives upto the third order difference term.**

Sol: **(A.U 2014,M/D2010,2011)**

$\left(\frac{dy}{dx}\right)=\frac{1}{h}\left[\nabla y_n+\frac{\nabla^2}{2}y_n+\frac{\nabla^3}{3}y_n+\cdots\right]$

$\left(\frac{d^2y}{dx^2}\right)=\frac{1}{h^2}\left[\nabla^2 y_n+\nabla^3 y_n+\frac{11}{12}\nabla^4 y_n+\cdots\right]$

$\left(\frac{d^3y}{dx^3}\right)=\frac{1}{h^3}\left[\nabla^3 y_n+\frac{3}{2}\nabla^4 y_n+\cdots\right]$

3.**Why is Trapezoidal rule so called** ? **(A.U 2010,2013,2012**)

Sol: The trapezoidal rule is so called,because it approximates the integral by the sum of n trapezoids.

4.**When does simpson's rule give exact result** ? **(A.U2006,2007,2011)**

Sol: Simpson's rule will give exact result,if the entire curve y =f(x) is itself a parabola.

5.**State Simpson's three eighth rule.** **(A.U 2010,2011,2014,2015)**

Sol:

$\int_a^b y\,dx=\frac{3h}{8}\left[(y_1+y_{n+1})+3(y_2+y_{5+\cdots+}y_n)+2(y_4+y_7+\cdots+y_{n-2}\right]$

6.**What is the order of error in trapezoidal formula.** **(A.U 2010,2011,2014)**

Sol:

Error in the trapezoidal formula is of the order h^2.

7.**What is the order of error in Simpson's formula** ? **(A.U 2007,2010,2011,2012,2015)**

Sol:

Error in the Simpson's formula is of the order h^4.

8. **What are the errors in trapezoidal and Simpson's rules of numerical integration**? **(A.U 2010)**

Sol:

Error in Trapezoidal rule $|E|< \frac{(b-a)}{12}$ h^2 M

Error in Simpson's rule $|E|< \frac{(b-a)}{180}$ h^4 M

M in the interval (a, b), h = $\frac{(b-a)}{n}$

9.**When do you apply Simpson's 1/3 rule** ? **(A.U2011,2012)**

Sol:

The interval of integration must be divided into an even number of subintervals of width h.

10.**Using Simpson's rule find $\int_0^4 e^x\,dx$,given e^0=1,e^1=2.72,e^2=7.39,e^3=20.09 and e^4=54.6** **(A.U 2012)**

Sol:

Let y=e^x and h=1,by Simpson's rule

$$\int_0^4 e^x\,dx = \frac{h}{3}[(y_0 + y_4) + 2(y_2) + 4(y_1 + y_3)]$$

$$= \frac{1}{3}[(1 + 54.6) + 2(7.39) + 4(2.72 + 20.09)]$$

$$= 53.8733$$

11. **Evaluate I =$\int_0^1 \frac{dx}{1+x}$ by Gaussian formula with two points.** **(A.U2009,2012)**

Sol:

Let f(x) = $\frac{1}{1+x}$

Let x= $\frac{b-a}{2}$ z +$\frac{b+a}{2}$ Here a=0,b=1

$\therefore$ x = $\frac{1-0}{2}$ z +$\frac{1+0}{2}$

x = $\frac{1}{2}$ z + $\frac{1}{2}$;x=0→ z= -1

$dx = \frac{1}{2} dz$; x=1→ z= 1

$\therefore I = \int_{-1}^{1} \frac{\frac{1}{2} dz}{1+(\frac{1}{2} z + \frac{1}{2})} = \frac{1}{2} \int_{-1}^{1} \frac{dz}{\frac{3}{2}+\frac{z}{2}}$

$= \int_{-1}^{1} \frac{dz}{z+3} = f[\frac{-1}{\sqrt{3}}] + f[\frac{1}{\sqrt{3}}]$

Here $f(z) = \frac{1}{z+3}$

$f[\frac{-1}{\sqrt{3}}] = \frac{1}{\frac{-1}{\sqrt{3}}+3} = 0.41277$

$f[\frac{1}{\sqrt{3}}] = \frac{1}{\frac{1}{\sqrt{3}}+3} = 0.27954$

∴ I=0.41277+0.27954=0.6923

12.**State three point Gaussian Quadrature formula.** **(A.U 2004,2012,2014)**

Sol:

Three points Gaussian Quadrature formula is

$\int_{-1}^{1} f(x)dx = \frac{5}{9} (f[\sqrt{\frac{-3}{5}}] + f[\sqrt{\frac{3}{5}}]) + \frac{8}{9} f(0)$

13.**State Trapezoidal rule for evaluating $\int_a^b \int_c^d f(x,y)dxdy$** **(A.U20112014)**

Sol:

$I=\frac{hk}{4}[(sum\ of\ values\ of\ f\ at\ the\ four\ corners) +$
$2(sum\ of\ values\ of\ f\ at\ the\ remaining\ nodes\ on\ the\ boundary) +$
$4(sum\ of\ the\ values\ of\ f\ at\ the\ interior\ nodes)]$

UNIT-IV

INITIAL VALUE PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS

1.**State the disadvantage of Taylor's method.** **(A.U2011,2012,2014)**

Sol:

In the differential equation $\frac{dy}{dx} = f(x, y)$, the function f(x,y) may have a complicated algebraic structure. Then the evaluation of higher order derivatives may become tedious. This is the demerit of this method .

2.**What is the truncation error of Taylor's method** ? **(A.U 2010,2013)**

Sol:

$E_n = \frac{h^{n-1}}{(n+1)!} f^{(n+1)} =(x_i +\theta h)$

3.**In solving $\frac{dy}{dx} = f(x, y)$,y(x_0) = y_0 ,write down Taylor's series for y(x_1).** **(AU2010)**

Sol:

$y(x_1) = y_0 + \frac{h}{1!} y_0' + \frac{h^2}{2!} y_0'' + \ldots\ldots.$

4.**Write Taylor's formula to solve y' = f(x,y) with y(x_0) = y_0.** **(A.U 2010)**

Sol:

$y = y_0 +(x-x_0) y_0' +\frac{(x-x_0)^2}{2!} y_0'' + \ldots\ldots\ldots$

5.**State modified Euler algorithm to solve y' = f(x,y), y(x_0) = y_0 at x= x_0+h.**

Sol: **(A.U 2010,2011,2012,2013)**

$Y_{n+1} = y_n +hf\left[x_n + \frac{h}{2} , y_n + \frac{h}{2} f(x_n, y_n\right]$

$Y_1 = y_0 + hf\left[x_0 + \frac{h}{2} , y_0 + \frac{h}{2} f(x_0, y_0\right]$

6.**Write down the Runge-Kutta formula of fourth order to solve $\frac{dy}{dx} = f(x, y)$,with y(x_0) = y_0.** **(AU2010,2012,2014)**

Sol:

Let h denote the interval between equidistant values of x.if the initial values are (x_0, y_0).

$K_1 = h f(x_0, y_0)$

$K_2 = h f(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$

$K_3 = h f(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$

$K_4 = h f(x_0 + h, y_0 + k_3)$

And $\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

7.**State the special advantage of Runge-Kutta method over Taylor method (AU 2010)**

(OR)

Compare Taylor's series and Runge-Kutta method. (AU 2011,2012)

Sol:

Runge-Kutta methods do not require prior calculation of higher derivatives of y(x) as the Taylor methos does.

Since,the differential equations using in applications often complicated,the calculation of derivatives may be difficult.

Also the Runge- Kutta formulas involve the computation of f (x,y) at various positions, instead of derivatives and this function occurs in the given equation.

8.**What is the error in Milne's Predictor formula. (AU 2012)**

Sol:

The correct term is $\frac{14h}{45} \Delta^4 y_0'$.

9. **Write down Adam's –Bashforth predictor and corrector formulae (A.U2004, 2005, 2009, 2010, 2011,2012,2014)**

Sol:

$Y_{k+1,p} = y_k + \frac{h}{24}(55 y_k' - 59 y_{k-1}' + 37 y_{k-2}' - 9 y_{k-3}')$

$Y_{k+1,c} = y_k + \frac{h}{24}(9 y_{k+1}' + 19 y_k' - 5 y_{k-1}' + y_{k-2}')$

10.**How many prior values are required to predict the next value in Adam's method**? **(AU2006)**

Sol : Four prior values.

11.**Give the multistep methods available for solving ordinary differential equation.**

Sol: (AU2007,2010)

1.Euler's Method

2. Runge- Kutta Method

3. Milne's Method

4. Adam's –Bashforth Method

UNIT-V

BOUNDARY VALUE PROBLEMS IN ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

1. State Standard five point formula. **(A/M 2015,N/D2014)**

Sol:

$$u_{i,j} = \frac{1}{4}\left[u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1}\right]$$

2.Write down the implicit formula to solve one dimensional heat flow equation $u_{xx} = \frac{1}{c^2} u_r$.

Sol: **(A/M 2011,/N/D2011)**

$$u_{i,j+1} = \frac{1}{4}\left[u_{i-1,j} + u_{i+1,j} + u_{i+1,j+1} + u_{i-1,j+1}\right]$$

3.Write down the Bender-Schmidt recurrence relation for one dimensional heat equation? **(A/M 2011,N/D2012)**

Sol:

The Bender-Schmidt recurrence relation for one dimensional heat equation is

$$u_{i,j+1} = \frac{1}{2}\left[u_{i+1,j} + u_{i-1,j}\right] \quad \text{, if } \lambda = \tfrac{1}{2}$$

$$u_{i,j+1} = \lambda u_{i-1,j} + \lambda u_{i+1,j} + (1-2\lambda)u_{i,j}$$

4.Write down the error for solving Laplace and Poisson's equations by finite difference method? **(A/M 2011)**

Sol:

The error in replacing $\frac{\partial^2 u}{\partial x^2}$ by the difference expression is of the order o (h^2).

Since h = k, the error is replacing $\frac{\partial^2 u}{\partial y^2}$ by the difference expression is of the order o(h^2)

Hence the error for solving Laplace and Poisson o (h^2).

For SFPF

$$T.E = \frac{h^4}{12}\left[\frac{\partial^4 u}{\partial x^4} + \frac{\partial^4 u}{\partial y^4}\right]_{i,j} + \ldots\ldots\ldots\ldots$$

For DFPF

$$T.E = \frac{h^4}{6}\left[\frac{\partial^4 u}{\partial x^4} + 6\frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^4 u}{\partial y^4}\right]_{i,j} + \ldots\ldots\ldots\ldots$$

5.State the implicit finite difference scheme for finite dimensional heat equation?

Sol: **(A/M 2012)**

$$\frac{\lambda}{2}u_{i+1,j+1}-(\lambda+1)u_{i,j+1}+\frac{\lambda}{2}u_{i-1,j+1}=-\frac{\lambda}{2}u_{i+1,j}+(\lambda-1)u_{i,j}-\frac{\lambda}{2}u_{i-1,j},\lambda=\frac{k}{ah^2}$$

6.Write down the one dimensional wave equation and the boundary conditions?

$$\frac{\partial^2 u}{\partial t^2}=\alpha^2\frac{\partial^2 u}{\partial x^2}$$ **(A/M 2012)**

Sol:

(i)u(0.t)=0

(ii)u(l,t)=0, for all t > 0

iii)u(x,0)=f(x),0<x<l

(iv) $\frac{\partial u}{\partial t}$ (x,0)=0 , 0<x<l.

7.Name at least two numerical methods that are used to solve one dimensional diffusion equation? (A/M 2011)

Sol:

Two numerical methods to solve one-dimensional diffusion equation.

(1)Bender-Schmidt method

(2)Crank-Nicholson method

8.Write down the finite difference scheme for solving Poisson's equation? (A/M 2013)

Sol:

$$u_{i-1,j}+u_{i+1,j}+u_{i,j-1}+u_{i,j+1}-4u_{i,j}=h^2 f(ih, jh)$$

9.Express $a^2u_{xx}=u_{tt}$ in terms of difference quotients? (A/M 2011)

Sol:

The general form of the difference equation to solve the equation $u_{tt}=a^2u_{xx}$ is

$u_{i,j+1}=\lambda^2a^2(u_{i-1,j}+u_{i+1,j})+2(1-\lambda^2a^2)u_{i,j}-u_{i,j-1}$ -------- (1) (wave equation)

If $\lambda^2a^2 = 1$ coefficient of $u_{I,j}$ in equation (1) is = 0.

The recurrence equation (1) takes The simplified form

$$u_{i,j+1}= u_{i-1,j}+u_{i+1,j}-u_{i,j-1}$$

10.Write a note on the stability and convergence of the solution of the difference equation corresponding to the hyperbolic equation $u_{tt}=a^2u_{xx}$? (A/M 2013)

Sol:

For λ=1/a, the solution of the difference equation is stable and coincide with the solution of the differential equation

For λ>1/a, the solution is unstable.

For λ<1/a, the solution is stable but not convergent

11.Write down the diagonal five point formula in Laplace equation? (A/M 2011,2013,2014)

Sol:

$$u_{i,j}=\frac{1}{4}\left[u_{i-1,j-1}+u_{i-1,j+1}+u_{i+1,j-1}+u_{i+1,j+1}\right]$$

12.Explain the terms initial and boundary value problems? (A/M 2012,2010)

Sol:

INITIAL VALUE PROBLEM:

In solving a differential equation analytically,we usually find a general solution containing arbitrary constants and then evaluate the arbitrary constants . so that the expression agrees with the initial conditions, the problem is called initial value problem.

BOUNDARY VALUE PROBLEM:

When the differential equation is to be solved satisfying the conditions specified at the end points of an interval , the problem is called boundary value problem.

13.**State whether the crank-nicoloson's is an explicit or implicit scheme.justify(M/J 2014)**

Sol:

It is an implicit scheme.The values of the temperature u at theinterior points are obtained by solving a system of equations at each step.

******_ALL THE BEST_******