UNIT – I LOGIC AND PROOFS

PART – A

1. **What are the contrapositive, the converse and the inverse of the conditional statement , " If there is a rain ,then I buy an umbrella** **[Nov/Dec -2016]**

Answer:

Contrapositive: $\neg q \rightarrow \neg p$

That is "If I don"t buy an umbrella, then there is no rain"

Converse: $q \rightarrow p$

That is "If I buy an umbrella , then there is rain"

Inverse: $\neg p \rightarrow \neg q$

That is " If there is no rain ,then I will not buy an umbrella"

2. **Express $A \leftrightarrow B$ in terms of the connectives $\{\wedge, \neg\}$** **[May/June-2016]**

Answer:

$A \leftrightarrow B \Leftrightarrow (\neg A \vee B) \wedge (\neg B \vee A) \Leftrightarrow \neg(A \wedge \neg B) \wedge \neg(B \wedge \neg A)$

3. **Find a counter example, if possible, to these universally quantified statements, whose the universe of discourse for all variables consists of all integers.** **[Nov/Dec -2014]**

Answer:

(a) $\forall x \ \forall y(x^2 = y^2 \rightarrow x = y)$**.**

(b) $\forall x \ \forall y(xy \geq x)$**.**

Answer:

(a) and (b)

4. **Construct the truth table** $P \rightarrow Q$ **[May/June -2016]**

Answer:

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

5. **Show that $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$ without using truth tables.**

Answer:

$$p \rightarrow (q \rightarrow r) \Leftrightarrow \neg p \vee (\neg q \vee r) \Leftrightarrow (\neg p \vee \neg q) \vee r \Leftrightarrow \neg (p \wedge q) \vee r \Leftrightarrow (p \wedge q) \rightarrow r$$

6. **Show that $(\neg p) \rightarrow (p \rightarrow q)$ is a tautology**.

Answer: $(\neg p) \rightarrow (p \rightarrow q) \Leftrightarrow p \vee (\neg p \vee q) \Leftrightarrow (p \vee \neg p) \vee q \Leftrightarrow T \vee q \Leftrightarrow T$

7. **Write the truth table for the formula** $(p \rightarrow q) \leftrightarrow (\neg p \rightarrow \neg q)$. **[April/May- 2015]**

Answer:

P	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg p \rightarrow \neg q$	$(p \rightarrow q) \leftrightarrow (\neg p \rightarrow \neg q)$.
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

8. **What are the negation of the statements $\forall x(x^2 > x)$ *and* $\exists x(x^2 = 2)$?** **[Nov/Dec-2013]**

Answer:

The negation of $\forall x(x^2 > x)$ is $\neg\forall x(x^2 > x)$

$\Leftrightarrow \exists x\neg(x^2 > x)$

$\Leftrightarrow \exists x(x^2 \leq x)$

The negation of $\exists x(x^2 = 2)$ *is* $\neg\exists x(x^2 = 2)$

$\Leftrightarrow \forall x\neg(x^2 = 2)$

$\Leftrightarrow \forall x(x^2 \neq 2)$

9. **Rewrite the following using quantifiers "Every student in the class studied calculus".**

Answer:

Let is a student and studied calculus

Symbolic form $\forall x\big(P(x) \rightarrow Q(x)\big)$

10. **Write in symbolic form "If you work hard, then you will be rewarded"** **[Nov/Dec- 2015]**

Answer:

Let p: „you work hard" and q: „you will be rewarded"

Symbolic Form: $p \rightarrow q$

11. **When do you say that two compound propositions are equivalent?**

Answer:

Two statements A and B are equivalent if and only if $A \leftrightarrow B$ is a tautology. It is denoted by the symbol $A \Leftrightarrow B$ which is read as "A is equivalence to B"

12. **Prove that $(p \leftrightarrow q) \Leftrightarrow (p \wedge q) \vee (\neg p \wedge \neg q)$**

Answer:

$(p \leftrightarrow q) \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p) \Leftrightarrow (\neg p \vee q) \wedge (\neg q \vee p)$

$\Leftrightarrow (\neg p \wedge \neg q) \vee (\neg p \wedge p) \vee (q \wedge \neg q) \vee (p \wedge q)$

$\Leftrightarrow (\neg p \wedge \neg q) \vee (p \wedge q)$

13. **Give an indirect proof of the theorem "If (3x + 2)) is odd then x is odd".**

Answer:

Assume the conclusion of this implication is false.

That is, x is even ($x = 2l$, for some l) which implies $3x + 2 = 3(2l) + 2 = 2(3l + 1)$, even

Negation of the conclusion of the implication implies that the hypothesis is false.

Hence the implication is true.

14. **Given P = {2,3,4,5,6} state the truth value of the statement ($\exists x \in P$) ($x + 3 = 10$)** **[April/May- 2015]**

Answer:

P = {2,3,4,5,6}, None of the values taken from the set P satisfies the Equation ($x + 3 = 10$) $\therefore$ Truth value of the quantified statement is false.

15. **Write the statement in symbolic form "Some real numbers are rational".**

Answer:

Let is a real number and is rational

Symbolic form: $\exists x\big(R(x) \wedge Q(x)\big)$.

16. **Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent.** **[Nov/Dec- 2014]**

Answer:

For **$(p\to r)\wedge(q\to r)$** to be false, one of the two implications must be false, which happens exactly when r is false and at least one of p and q is true, but these are precisely the cases in which $p \vee q$ is true and$r$is false. Which is precisely when($p\vee q)\to r$ is false. Since the twopropositions are false in exactly the same situations they are logically equivalent.

17. **What is the duality law of logical expression? Give the dual of** $(P\vee F)\wedge(Q\vee T)$

Answer:

In an expression, if we replace $\wedge,\vee,T, F$ respectively by $\vee,\wedge,F, T$ the resulting new formula is the dual of the given expression.

$\therefore$ Dual of the given formula: $(P\wedge T)\vee(Q\wedge F)$

18. **Let E= {-1, 0, 1, 2} denote the universe of discourse. If** $P(x,y) :x+y=1$, **find the truth value of** $(\forall x)(\exists y)\ p(x, y)$. **[May/June-2014]**

Answer:

$(\forall x)(\exists y)\ p(x, y)$ denotes the proposition "For every real number x there is a real number y such that$P(x,y)$."

The statement $(\forall x)(\exists y)p(x,y)$ is true for this universe since -1+2=1,0+1=1

19. **Define Contradiction.**

Answer:

A propositional formula which is always false irrespective of the truth values of the individual variables is a contradiction.

20. **Define Universal quantification and Existential quantification. Answer:**

The Universal quantification of a predicate formula $P(x)$ is the proposition, denoted by $\forall xP(x)$ that is true if P (a) is true for all subject a.

The Existential quantification of a predicate formula $P(x)$ is the proposition, denoted by $\exists xP(x)$ that is true if P(a) is true for some subject a.

PART-B

1. Show that $\neg(p\to q)$ and $p\wedge\neg q$ are logically equivalent
2. Show that $\neg(p\vee(\neg p\wedge q))$ is logically equivalent to $\neg p\wedge\neg q$
3. Show that $(p\wedge q)\to(p\vee q)$ is a tautology
4. Show that $(\neg p\wedge(\neg q\wedge r))\vee(q\wedge r)\vee(p\wedge r)\equiv r$
5. State and explain the proof methods.
6. Prove that the following argument is valid: $p\to\neg q, r\to q, r \Rightarrow \neg p$
7. Without using truth table find the PDNF and PCNF of $P\to(Q\Lambda R)\Lambda(\neg P\to(\neg Q\Lambda\neg R))$
8. Prove that the premises $a\to(b\to c),\ d\to(b\Lambda\neg c)$ *and* $(a\Lambda d)$ are inconsistent.
9. Prove that $\forall x(P(x)\to Q(x)),\ \forall x(R(x)\to\neg Q(x)) \Rightarrow \forall x(R(x)\to\neg P(x))$.
10. Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.
11. Verify the validity of the following argument: Every living thing is a plant or an animal. John's gold fishis alive and it is not a plant. All animals have hearts. Therefore John's gold fish has a heart.
12. Show that $(P\to Q)\Lambda(R\to S),(Q\Lambda M)\Lambda(S\to N),\neg(M\Lambda N)$ *and* $(P\to R)\Rightarrow\neg P$.
13. Obtain the principal disjunctive normal form and principal conjunctive normal form of the statement $p\vee(\neg p\to(q\vee(\neg q\to r)))$
14. Use indirect method to prove that $(\forall x)(\ p(x)\vee q(x)) \Rightarrow (\forall x)\ p(x)\vee(\exists x)q(x)$.

15. Determine the validity of the following argument. If 7 is less than 4, then 7 is not prime number.7 is not less than 4, therefore 7 is a prime number.

16. Show $J \wedge S$ logically from the premises $P \rightarrow Q, Q \rightarrow \neg R, R, P \vee (J \wedge S)$.

17. Show that $R \rightarrow \neg Q, R \vee S, S \rightarrow \neg Q, P \rightarrow Q, P$ are inconsistent.

18 Give a direct proof for $p \rightarrow (q \rightarrow s), (\neg r \vee p), q \Rightarrow (r \rightarrow s)$.

UNIT II COMBINATORICS

PART – A

1. **Use Mathematical induction to show that 1+2+3+…+n = $\frac{n(n+1)}{2}$** **[April/May- 2015]**

Answer:

Let P(n) : 1+2+3+…+n= $\frac{n(n+1)}{2}$

Basis: P(1): 1= $\frac{1(1+1)}{2}$ = 1, ∴P(1) is true.

Induction: Assume that P(k) is true, i.e. 1+2+3+…+k= $\frac{k(k+1)}{2}$

Consider P(k+1)

Now 1+2+3+…+ k + (k+1)= $\frac{k(k+1)}{2}$ + k+1 = $\frac{(k+1)(k+2)}{2}$ = $\frac{(k+1)((k+1)+1)}{2}$

∴ P(k+1) is true i.e. P(k) ⇒ P(k+1)

∴ By induction, 1+2+3+…+n= $\frac{n(n+1)}{2}$

2. **State pigeon hole principle.** **[May/June- 2016]&[Nov/Dec 2015]&[Nov/Dec 2013]**

Answer:

If (n+1) pigeons occupy n holes then at least one hole has more than 1 pigeon.

3. **Show that, among 100 people, at least 9 of them were born in the same month.**

Answer:

Here no. of pigeon = m = no. of people = 100

No. of holes = n = no. of month = 12

Then by generalized pigeonhole principle, $\left\lceil \frac{100-1}{12} \right\rceil + 1 = 9$ were born in the same month.

4. **Find the number of non-negative integer solutions of the equation** $x_1+x_2+x_3=11$.

[May/June- 2012]

Answer:

Let n = Total number of solution.

$= C(n+r-1,r) = C(3+11-1,11) = C(13,11) = {}^{13}C_{11}=78$

5. **How many permutations of the letters in ABCDEFGH contain the string ABC Answer:**

Because the letters ABC must occur as block, we can find the answer by finding no of permutation of six objects, namely the block ABC and individual letters D, E, F, G and H. Hence, there are 6! =720 permutations of the letters in ABCDEFGH which contains the string ABC.

6. **How many different bit strings are there of length 7?** **[Nov/Dec 2013]**

Answer:

By product rule, $2^7=128$ ways

7. **How many ways are there to form a committee, if the committee consists of 3 educationalists and 4 socialist, if there are 9 educationalists and 11 socialist?**

Answer:

The 3 educationalist can be chosen from 9 educationalists in $9c_3$ ways.

The 4 socialist can be chosen from 11 socialist in $11C_4$ ways.

By product rule, the no of ways to select, the committee is = $9C_3.11C_4$ = 27720 ways.

8. **There are 5 questions in a question paper in how many ways can a boy solve one or more questions?**

Answer:

The boy can dispose of each question in two ways. He may either solve it or leave it.

Thus the number of ways of disposing all the questions= 2^5

But this includes the case in which he has left all the questions unsolved.

The total no of ways of solving the paper = 2^5-1 = 31.

9. **What is well ordering principle?** **[May/June 2014]**

Answer:

Every non empty set of non negative integers has the least element

10. **Twelve students want to place order of different ice-creams in a ice-cream parlour, which has six type of ice-creams. Find the number of orders that the twelve students can place**

[May/June-2013]

Answer:

Here n=12, r = 6 the no of possible selection is ${}^{12+6-1}C_6={}^{17}C_6$ = 12376 possibilities.

11. **Find the recurrence relation whose solution is $S(n)=a^n, n\geq 1$** **[Nov/Dec 2016]**

Answer:

Given $S(n)=a^n \Rightarrow S(n-1)=a^{n-1}=a^n/a$

$\Rightarrow S(n)=aS(n-1)$ is the required recurrence relation.

12. **Find the associated homogeneous solution for $a_n=3a_{n-1}+2n$.**

Answer:

Its associated homogeneous equation is

$a_n-3a_{n-1}=0$ Its characteristic equation is r-3 =0 $\Rightarrow$ r =3

Now, the solution of associated homogeneous equation is $a_n=A.3^n$

13. **Solve $S(k)-7S(k-1)+10S(k-2)=0$**

Solution:

The associated homogeneous relation is $S(k)-7S(k-1)+10S(k-2)=0$

Its characteristic equation is $r^2-7r+10=0 \Rightarrow (r-2)(r-5)=0 \Rightarrow$ r =2,5 The solution of associated homogeneous equation is $S_k=A.2^k+B.5^k$

14. **Define Generating function.**

Answer:

The generating function for the sequence *'s'* with terms $a_0,a_1,.........a_n$ …, of real numbers is the infinite sum . $G(x) = a_0+a_1x+.........+a_nx^n+.....$ $=\sum_{n=0}^{\infty} a_n x^n$.

15. **Find the generating function for the sequence „s" with terms 1, 2, 3, 4, …**

Answer:

$$G(x)=G(s,x)=\sum_{n=0}^{\infty}(n+1)x^n = 1+2x+3x^2+.......... = (1-x)^{-2}= \frac{1}{(1-x)^2}.$$

16. **How many permutations of (a, b, c, d, e, f, g) end with a?** **[Nov/Dec- 2014]**

Answer:

$6! \times 1! = 720$

17. **How many different words are there in the word ENGINEERING** **[May/June -2016]**

Answer:

Number of arrangements = $\frac{11!}{3!3!2!2!}$

18. **State the principle of strong induction.** **[May/June -2014]**

Answer:

Let P(n) be a proposition . The principle includes two steps Step 1 (Basis step) P(1) is true

Step 2 (induction step) If P(j) is true for j = 2,3, …, k then P(k+1) is true

i.e. [$p(2) \wedge p(3) \wedge \ldots \wedge p(k)$] $\rightarrow$ P(k+1) is true for every positive integers k.

19. **How many students must be in a class to guarantee that atleast two students receive the same score on the final exam if the exam is graded on a scale from 0 to 100 points?**

Answer:

There are 101 possible scores as 0, 1, 2, … , 100.

By Pigeon hole principle, we have among 102 students there must be at least two students with the same score.

The class should contain minimum 102 students.

Find the recurrence relation for the Fibonacci sequence. **[May/June 2012]**

Answer:

The recurrence relation corresponding to the Fibonacci sequence $\{f_n\}$, $n \geq 0$ is

$f_{n+2} = f_{n+1} + f_n$, $n \geq 0$ with initial conditions $f_0 = 0$, $f_1 = 1$

PART B

1. Prove by induction: $1+2+2^2+2^3+2^4+ . . . + 2^n = 2^{n+1} - 1$
2. Prove by mathematical induction $\forall n \geq 1$, $n^3 + 2n$ is a multiple of 3.
3. Use mathematical induction and show that $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$ for $n \geq 1$.
4. Prove that $8^n - 3^n$ is a multiple of 5 by using method of induction.
5. Using induction prove that $1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$.
6. A question paper has 3 parts, part –A , part-B and part –C having 12 , 4 and 4 questions respectively .A student has to answer 10 questions from part-A and 5 questions from part-B and part-C put together selecting at least 2 from each one of these two parts .In how many ways the selection of question can be made?
7. In a survey of 100 students , if was found that 40 studied mathematics , 64 studied physics 35 studied chemistry , 1 studied all 3 subjects , 25 studied maths and physics , 3 studied math's and chemistry and 20 studied physics and chemistry . Find the number of students who studied chemistry only.
8. State the generalized pigeonhole principle .Using this, find the minimum number of students in a class to be sure that at least 3 of them are born in the same month.
9. A bit is either 0 or 1 .A byte is a sequence of 8 bits .find the number of bytes. Among these how many are
   (i) Starting with 11 and ending with 00
   (ii) Starting with 11 but not ending with 00 or not starting with 11 but ending with 00?
10. During a month with 30 days , a base ball team plays at least one game a day , but no more than 45 games .Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games.
11. Show that for every integer n there is a multiple of n that has only 0s and 1s in its decimal expansion.
12. Show that among any n+1 positive integers not exceeding 2n there must be an integer that divides one of the other integers.
13. Every sequence of n^2+1 distinct real numbers contains a subsequence of length n+1 that is either strictly increasing or strictly decreasing.
14. Find the number of primes not exceeding 100.
15. How many solutions does $x_1 + x_2 + x_3 = 11$ have, where x_1 , x_2 ,x_3 are non negative integers with $x_1 \leq 3, x_2 \leq 4$ and $x_3 \leq 6$?
16. How many positive integers less than 10, 00,000 have the sum of their digits equal to 19?
17. How many positive integers n can be formed using the digits 3 ,4 , 4 ,5 ,5 ,6 ,7 if n has to exceed 5000000?
18. Find the number of integers between 1 and 250 both inclusive that are
   (i) Divisible by any of the integers 2, 3, 5, 7. (ii) Not divisible by any of these integers.
19. For $m \in Z^+$ and m odd, prove that there exists a positive integer n such that m divides $2^n - 1$.

20. Show that if any 11 numbers are chosen from the set {1, 2,…,20} then one of them will be a multiple of another.

21. Use the generating function to solve the recurrence relation
$a_{n+2} – 8a_{n+1} + 15a_n = 0$ given that $a_0 = 2$, $a_1 = 8$

22. Using generating function, solve the difference equation
$y_{n+2} – y_{n+1} – 6 y_n = 0$, $y_1 = 1$, $y_0 = 2$.

23. Solve: S(k) – 4 S(k- 1) + 4 S(k-2) = 3k + 2k, S(0) = 1 , S(1) = 1

24. Find the generating function of Fibonacci sequence

25. Solve S(n) – 2 S(n-1) – 3 S(n-2) =0 , $n \geq 2$ with S(0) = 3 and S(1) = 1 by using generating function .

26. Solve Y (n) -7 Y (n-1) + 10 Y (n-2) = 6 + 8n with Y (0) = 1, Y (1) =2.

27. Solve the recurrence relation $a_n = 2(a_{n-1} – a_{n-2})$,
where $n \geq 2$ and $a_0 = 1$, $a_1 = 2$.

28. Using the generating function solve $a_n – 3 a_{n-1} = n$, $n \geq 1$, a0 = 1

29. Solve the following recurrence relation a n+2 – 2 a n+1 + an = 2n
with $a_0 = 2$, $a_1 = 1$ using generating functions.

30. Solve the recurrence relations S (k) – 4 S (k-1) -11 S (k-2) + 30 S (k-3) = 0,
S(0) = 0 , S(1) = -35 , s(2) = - 85.

31. Solve the recurrence relation of the Fibonacci sequence of numbers
$f_n = f_{n-1} + f_{n-2}$, n > 2 with the initial conditions $f_1 = 1$, $f_2 = 1$.

32. A box contains six white balls and five red balls. Find the number of ways five balls can be drawn from the box if (1) they can be any color (2) Two must be white (3) They must all be the same color.

33. Use mathematical induction to show that $3^n + 7^n – 2$ is divisible by 8 for all all n greater than orequal to 1

34. There are 250 students in an Engineering College. Of these 188 have taken a course in FORTRAN, 100 have taken a course in C and 35 have taken a course in JAVA. Further 88 have taken courses inboth FORTRAN and C. 23 have taken courses in both C and JAVA and 29 have taken courses in both FORTRAN and JAVA. If 19 of these students have taken all the three courses, how many ofthese 250 students have not taken a course in any of these three programming languages?

UNIT III GRAPHS

PART – A

01. **Define complete graph.** **(Nov/Dec – 2016) &(May/June – 2014)**

Answer:

A graph of „*n*‟ vertices having each pair of distinct vertices joined by an edge is called a complete graph and is denoted by K_n.

02. **How many edges are there in a graph with 10 vertices each of degree 5?** **(May/June – 2016)**

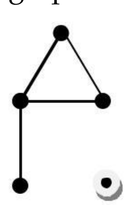
Answer:

$$\sum_i \deg(v_i) = 2e \Rightarrow 10 \times 5 = 2e \Rightarrow e = 25$$

3. **Does there exist a simple graph with five vertices of the 0, 1, 2, 2, 3 degrees? If so, draw such a graph.**

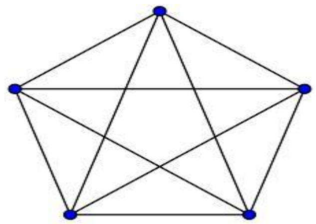
Answer:

Yes and the graph is



04. **Draw the complete graph K_5.** **(Nov/Dec – 2016)**

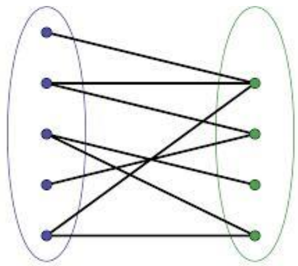
Answer:



5. **Define Bipartite Graph with**
example. Answer:

Let G = (V, E) be a graph. G is bipartite graph if its vertex set V can be partitioned into two nonempty disjoint subsets V_1 and V_2, called a bipartition, such that each edge has one end in V_1 and in V_2

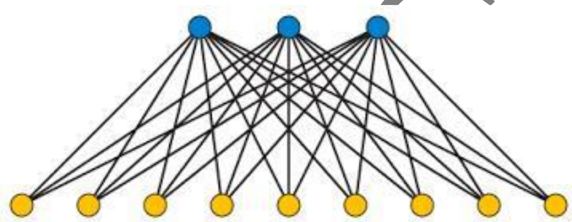
Example.



6. **Define complete bipartite graph with**
example Answer:

A complete bipartite graph is a bipartite graph with bipartition V_1 and V_2 in which each vertex of V_1 is joined by an edge to each vertex of V_2.

Example.



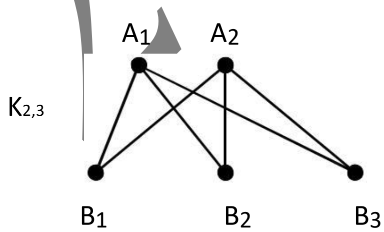
7. **Define Connected**
graph. Answer:

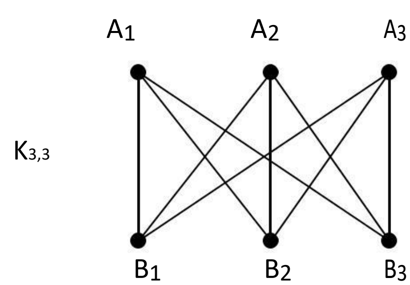
A graph for which each pair of vertices is joined by a trail is connected.

08. **State Handshaking theorem.** **(Nov/Dec – 2012)**
Answer:

If G = (V, E) is an undirected graph with e edges, then $\sum_i \deg(v_i) = 2e$

9. **Draw a complete bipartite graph of $K_{2,3}$ and $K_{3,3}$**
Answer:





10. **Define strongly connected**
graph. Answer:

A digraph G is said to be strongly connected if for every pair of vertices, both vertices of the pair are reachable from one another.

11. **Define adjacency matrix.**

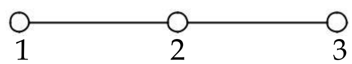
Answer:

Let G = (V, E) be a graph with n vertices. An $n \times n$ matrix A is an adjacency matrix for G if and only if for $\leq$ $\leq$, $A(i, j)= \begin{cases} 1 & \text{for } (i, j) \text{ in } E \\ 0 & \text{for} (i, j) \text{ is not in } E \end{cases}$

12. **Draw the graph represented by the given adjacency matrix.** **[Nov/Dec 2016]**

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Answer:



13. **Define Isomorphism of two graphs.** **(May/June 2014)**

Answer:

Two graphs $G_1 = (V_1,E_1)$ and $G_2 = (V_2,E_2)$ are the same or isomorphic, if there is a bijection $F: V_1 \to V_2$ such that $(u, v) \in E_1$ if and only if $(F(u), F(v)) \in E_2$.

14. **Define self complementary graph** **(May/June 2016)**

Answer:

The complement $\overline{G}$ of G is defined as a simple graph with the same vertex set as G and value two vertices u and v are adjacent in $\overline{G}$ only when they are not adjacent in G.

15. **Define spanning subgraph.**

Answer:

Let a graph $H = (V_1, E_1)$ is a subgraph of G = (V,E). H is a spanning subgraph of G if H is a subgraph of G with $V_1 = V$ and $E_1 \subset E$.

16. **Define Induced subgraph. Answer:**

A graph $H = (V_1, E_1)$ is a subgraph of G = (V,E). H is an induced subgraph of G such that E_1 consists of all the edges of G with both ends in V_1.

17. **Define Eulerian Circuit.**

Answer:

A circuit in a graph that includes each edge exactly once, the circuit is called an Eulerian circuit.

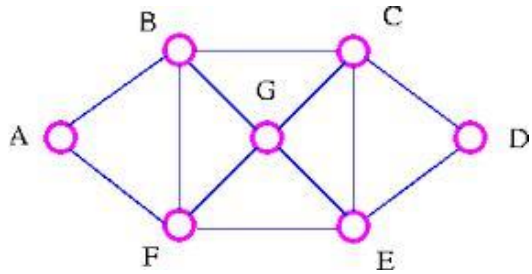
18. **State the condition for Eulerian cycle. Answer:**

(i) Starting and ending points are same.

(ii) Cycle should contain all edges of graph but exactly once

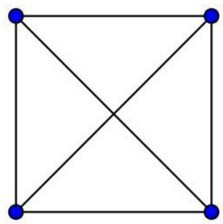
19. **Give an example of a graph which is Eulerian but not Hamiltonian.** **(April/May 2015)**

Answer:



20 **Give an example of non-Eulerian graph which is Hamiltonian.** **(May/June 2013)**

Answer:

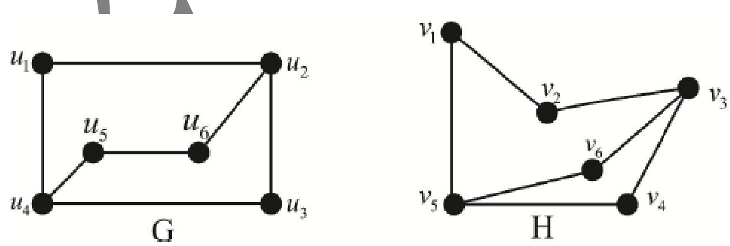


PART B

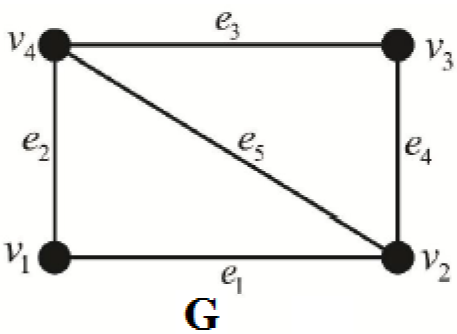
1. Draw the complete graph K_5 with vertices A,B,C,D,E. Draw all complete sub graph of K_5 with 4 vertices
2. Draw the graph with 5 vertices A, B, C, D, E such that deg (A) = 3, B is an odd degree vertex, deg(C) = 2 and D and E are adjacent.
3. The adjacency matrices of 2 pairs of graphs are as given below .Examine the isomorphism of G and H finding a permutation matrix

$$A_G = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, A_H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

4. Determine whether the graphs G and H are isomorphic

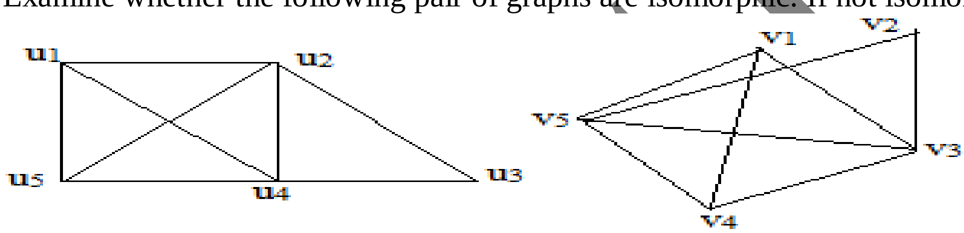


5. Define the degree of a vertex and prove that the number of vertices of odd degree is always even.
6. Prove that a simple graph with n vertices and k components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges.
7. Find the adjacency matrix of the following graph

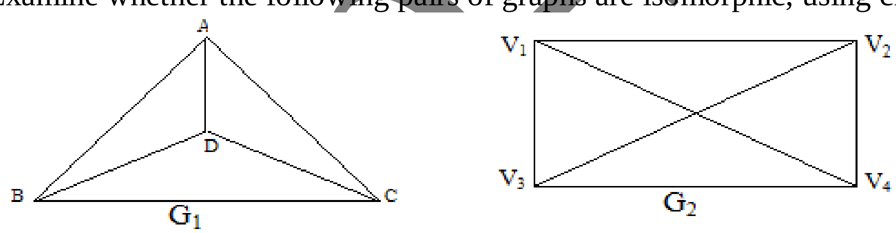


8. When a graph is said to be self-complementary? P.T if G is self complementary then it has 4n or 4n+1 vertices
9. Prove that there is a simple path between every pair of distinct vertices of a connected undirected graph.

10. Let G be a graph with adjacency matrix A with respect to the ordering v1,v2,……….vn , then prove that the number of different paths of length r from vi to vj equals (i,j) th entry of A^r.
11. Prove that a non empty connected graph is Eulerian if its vertices are all of even degree .
12. Prove if a connected multi-graph has an Euler circuit iff it has exactly 2 vertices of odd degree.
13. Show that K7 has Hamiltonian graph .How many edge disjoint Hamilton cycles are there in K7? List all the edge –disjoint Hamiltonian cycles. Is it Eulerian?
14. Examine whether the following pair of graphs are isomorphic. If not isomorphic, give the reasons



15. Examine whether the following pairs of graphs are isomorphic, using circuits



16. Let G be a simple undirected graph with n vertices. Let u and v be two non adjacent vertices in G such that deg (u) + deg (v) $\geq$ n in G. Show that G is Hamiltonian if and only if G + uv is Hamiltonian.
17. Define: (1) Adjacency matrix (2) Incidence matrix of a graph with examples.
18. Prove that if G is a simple graph with at least three vertices and $\delta(G) \geq \frac{|V(G)|}{2}$ then G is Hamiltonian.
19. If all the vertices of an undirected graph are each of degree, show that the number of edges of the graph is a multiple of k.
20. Show that the complete graph with n vertices K, has a Hamiltonian circuit whenever $n \geq 3$.

UNIT – IV ALGEBRAIC STRUCTURES
PART – A

1. **Define Algebraic system.**

Answer:

A system consisting of a set and one or more *n*-ary operations on the set will be called an algebraic system or simply algebra.

Example (Z, +) is an algebraic system.

02. **Define Semi Group.** **(Nov/Dec – 2016) &(Nov/Dec – 2014)**

Answer:

Let S be non empty set, * be a binary operation on S. The algebraic system (S, *) is called a semi group, if the operation is associative.

In other words (S,*) is a semi group if for any x, y, z $\in$ S, x* (y * z) = (x* y)* z.

3. **Define Monoid. (Nov/Dec – 2014)**

Answer:

A semi group (M, *) with identity element with respect to the operation * is called a Monoid.

In other words (M,*) is a Monoid if for any x, y, z $\in$ M, x * (y * z) = (x * y) * z and there exists an element e $\in$ M such that for any x $\in$ M then e * x = x * e = x.

4. **Give an example of semi group but not a Monoid.**

Answer:

The set of all positive integers over addition form a semi-group but it is not a Monoid.

5. **Define Group.**

Answer:

An algebraic system (G,*) is called a group if it satisfies the following properties:

(i) * is associative.

(ii) Identity element exists.

(iii) Inverse element exists.

6. **State any two properties of a group.**

Answer: (i) The identity element of a group is unique.

(ii) The inverse of each element is unique.

07. **Prove that identity element in a group is unique.** **(Nov/Dec – 2015) & (May/June - 2014)**

Answer:

Let (G,*) be a group.

Let „e_1" and „e_2" be the identity elements in G

Suppose e_1 is the identity, then

$e_1 * e_2 = e_2 * e_1 = e_2$

Suppose e_2 is the identity, then

$e_1 * e_2 = e_2 * e_1 = e_1$

Therefore $e_1 = e_2$.

Hence identity element is unique.

8. **Show that the inverse of an element in a group (G, *) is unique.**

Answer:

Let (G,*) be a group with identity element e.

Let „b" and „c" be inverses of an element „a"

$a * b = b * a = e$ also $a * c = c * a = e$.

$b = b * e = b * (a * c) = (b * a) * c = e * c = c$

$b = c$.

Hence inverse of an element is unique.

9. **Let Z be the group of integers with the binary operation * defined by $a*b=a+b-2$ for all $a, b \in Z$. Find the identity element of the group $\langle Z, *\rangle$** **(May/June 2016)**

Answer:

$a = a * e = a + e - 2 \Rightarrow e = 2$

10. **Show that every cyclic group is abelian.** **(May/June 2016)**

Answer:

Let (G,*) be an cyclic with „a" as generator

$\therefore \forall x, y \in G \Rightarrow x = a^m, y = a^n \therefore x * y = a^m * a^n = a^{m+n} = a^{n+m} = y * x$

11. **Prove that the semigroup homomorphism preserves idempotency**.

Answer:

Let $a \in S$ be an idempotent element.

$\therefore a*a = a$

$g(a * a) = g(a)$

$g(a) \circ g(a) = g(a)$

This shows that $g(a)$ is an idempotent element in T.

Therefore the property of idempotency is preserved under semigroup homomorphism.

12. **Define cyclic group.**

Answer:

A group (G,*) is said to be cyclic if there exists an element $a \in G$ such that every element of G can be written as some power of „a".

13. **Define group homomorphism with example. Answer:**

Let (G,*) and (S,) be two groups. A mapping f: G → S is said to be a group homomorphism if for any a, b ∈ G, f(a*b) = f(a) ∘ f(b).

Example:

Consider multiplicative group of positive real numbers $(R^{+},.)$ for any complex number u, the function $f_u: R^{+} \to C$ defined by $f_u(a)=a^u$ is a group homomorphism.

14. **Find the idempotent elements of $G=\{1,-1,i,-i\}$ under the binary operation multiplication. (Nov/Dec 2016) Answer:**

The idempotent element is e = 1

15. **Find the left cosets of $\{[0],[3]\}$ in the group $(Z_6,+)$ (April/May – 2015)**

Answer:

Let $Z_6=\{0,1,2,3,4,5\}$

$H=\{0,3\}$

$0+H=\{0,3\}=H$

$1+H=\{1,4\}$

$2+H=\{2,5\}$

$3+H=\{0,3\}=H$

$4+H=\{0,3\}=1+H$

$5+H=\{0,3\}=2+H$

0+H, 1+H and *2+H* are three distinct left coset of *H*.

16. **State Lagrange"s theorem. (Nov/Dec 2015)**

Answer:

The order of the subgroup of a finite group G divides the order of the group.

17. **Define Ring. (May/June - 2014)**

Answer:

An algebraic system (R, +, *) is called a ring if the binary operations + and R satisfies the following.

(i) (R,+) is an abelian group

(ii) (R,*) is a semi group

(iii) The operation is distributive over +.

18. **Define a Commutative ring. Answer:**

If the Ring (R, +, *) is commutative, then the ring (R, +, *) is called a commutative ring.

19. **Define field in an algebraic system. (April/May 2015)**

Answer:

A commutative ring (F, +, *) which has more than one element such that every nonzero element of F has a multiplicative inverse in F is called a field.

20. **Define Integral Domain.**

Answer:

A commutative ring R with a unit element is called an integral domain if R has no zero divisors.

PART B

1. Prove that for every row or column in the composition table of group <G,* > is a permutation of the elements of G.
2. Prove that in a group G the equations a*x = b and y*a = b have unique solutions for the unknowns x and y as $x=a^{-1} * b$ and $y = b * a^{-1}$ where $a, b \in G$.
3. Prove that the order of the subgroup of a finite group divides the order of the group.
4. Show that $R-\{0\}$ is an abelian group under $a*b=\frac{ab}{2}$.
5. If $f=\begin{pmatrix}1&2&3&4&5\\2&4&5&1&3\end{pmatrix}$ and $h=\begin{pmatrix}1&2&3&4&5\\5&2&4&3&1\end{pmatrix}$ are permutations on the set $A=\{1,2,3,4,5\}$, find a permutation g on A such that $f \circ g = h \circ f$.
6. Prove that the set of matrices $\begin{pmatrix}a&b\\-b&a\end{pmatrix}$ forms an abelian group with respect to matrix multiplication, where a & b are real numbers not both 0.
7. Prove that the order of the subgroup of a finite group divides the order of the group.
8. Show that R (set of real numbers)is an abelian group under $a*b=a+b+2ab$.
9. Prove that every finite group of order n is isomorphic to permutation group of order n.
10. If * is a binary operation on the set of real numbers defined by a* b = a + b + 2 a b, find (1) Identity element if exists (2) Inverse element if exists
11. Show that G is an abelian group iff $(a*b)^2=a^2*b^2$.
12. In any group, prove that the following (1) The identity element is unique (2) The inverse element is unique (3) $(a*b)^{-1}=b^{-1}*a^{-1}$.
13. Prove that the set of all matrices $\begin{bmatrix}a&b\\-b&a\end{bmatrix}$ forms an abelian under matrix product.
14. Show that group homomorphism preserves identity, inverse and subgroup.
15. State and prove Lagrange's theorem.
16. Let G be a group and a $\in$ G**.** Let $f:G\to G$ be given by $f(x)=axa^{-1}\ \forall x\in G$. Prove that f is an isomorphism of G on to G.
17. Show that the function f from of the permutation group P_n on to the multiplication group G = { -1 , 1 }defined by $f(a)=\begin{cases}1 & if\ a\ \ is\ even.\\-1 & if\ a\ \ is\ odd.\end{cases}$ is a homomorphism.
18. State and prove fundamental theorem of group homomorphism.

UNIT-V LATTICES AND BOOLEAN ALGEBRA

PART – A

1. **Define partial ordering on**

S. Answer:

A relation $\leq$ on a set S is called a partial ordering on S if it satisfies reflexive, anti-symmetric, transitive properties. A set S together with a partial ordering is called a partially ordered set or poset.

2. **In the poset(Z^+, /), are the integers 3 and 9 comparable? Are 5 and 7 comparable?**

Answer:

Since 3/9, the integers 3 and 9 are comparable. For 5, 7 neither 5/7 nor 7/5. Therefore, the integers 5 and 7 are not comparable.

03. **Define lattice.** **(Nov/Dec – 2016) & (Nov/Dec – 2015) & (May/June – 2014)**

Answer:

A partially ordered set (L,≤) in which every pair of elements has a least upper bound and greatest lower bound is called a lattice.

04. **Define lattice homomorphism and isomorphism.** **(April/May – 2015)**

Answer:

If $(L_1,\wedge,\vee)$ *and* $(L_2,\oplus,*)$ are two lattices, a mapping $f : L_1 \rightarrow L_2$ is called a lattice homomorphism from L_1 to L_2, if for any $a,b \in L_1$,

$f(a \vee b) = f(a) \oplus f(b)$ *and* $f(a \wedge b) = f(a) * f(b)$.

If a homomorphism $f: L_1 \rightarrow L_2$ of two lattices $(L_1,\wedge,\vee)$ *and* $(L_2,\oplus,*)$ is objective i.e. one -one, onto, then f is called an isomorphism.

5. **Define sub lattice with example.**

Answer:

A non-empty subset M of a lattice $(L,\wedge,\vee)$ is called a sub lattice of L, if and only if M is closed under both the operations $\wedge$ *and* $\vee$ that is if a, b∈M, then $a \vee b$ *and* $a \wedge b$ also in M. (S_n,D) is a sub lattice of (Z_+,D)

6. **Is the poset(Z^+, /) a lattice?**

Answer:

Let „a" and „b" be any two positive integer.

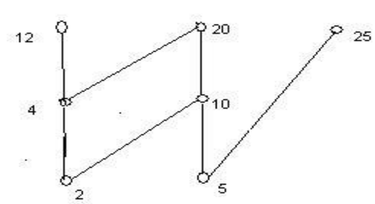
Then LUB {a,b} =LCM {a,b} and GLB {a,b}= GCD {a,b} should exists in Z^+ .
For example, let a=4, b=20
Then LUB{a, b} = LCM {4, 20} = 1 and GLB{a, b}=GCD{4, 20}=4

Hence, both GLB and LUB exist. Therefore The poset(Z^+, /) is a lattice.

7. **Which elements of the poset ({2, 4, 5, 10, 12, 20, 25}, /) are maximal and which are minimal?**

Answer:

The relation R is R={(2,4) (2,10) (2,12) (2,20) (4,12) (4,20) (5,10) (5,20) (5,25) (10,20)} Its Hasse diagram is



The maximal elements are 12, 20, and 25 and The minimal elements are 2 and 5.

8. **Give an example of a lattice which is a modular but not a distributive. Answer:**

Diamond lattice is a modular lattice but it is not a distributive lattice.

9. **Let X = {1,2,3,4,5,6} and R be a relation defined as** $\langle x, y \rangle \in R$ *iff* $x - y$ **is divisible by 3. Find the elements of the relation R** **(May/June – 2016)**

Answer:

$$R = \{\langle 1,1\rangle, \langle 2,2\rangle, \langle 3,3\rangle, \langle 4,4\rangle, \langle 5,5\rangle, \langle 6,6\rangle, \langle 1,4\rangle, \langle 4,1\rangle, \langle 2,5\rangle, \langle 5,2\rangle, \langle 3,6\rangle, \langle 6,3\rangle\}$$

10. **When is a lattice said to be bounded?** **(Nov/Dec – 2012)**

Answer:

The lattice is said to be bounded if it has both a lower bound and an upper bound.

11. **Prove that** $a+\bar{a}b=a+b$

Answer:

$$a+\bar{a}b = a + ab + \bar{a}b \qquad (a = a + ab)$$
$$= a + b(a + \bar{a})$$
$$= a + b$$

Since

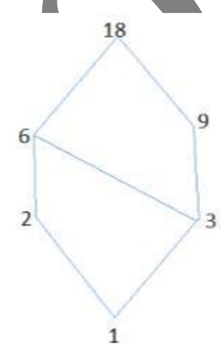
$b \wedge c = a$ *and* $b \vee c = 1$

$b \wedge a = a$ *and* $b \vee a = b$

Therefore b does not have any complement .the given lattice is not complemented lattice.

12. **Give an example of a lattice that is not complemented. (May /June – 2014) & (May/June – 2013)**

Answer:



13. **Prove that** $D_{42} \equiv \{S_{42}, D\}$ **is a complemented lattice by finding the complements of all the elements.**

Answer:

$$D_{42} = \{1, 2, 3, 4, 7, 14, 21, 42\}$$

The complement of 1 is 42, the complement of 2 is 21, the complement of 3 is 14, the complement of 6 is 7, the complement of 14 is 3, the complement of 21 is 2, the complement of 42 is 1, and the complement of 7 is 6. Every element has a complement. Hence $D_{42} \equiv \{S_{42}, D\}$ is a complemented lattice.

14. **Define a Boolean algebra.** **(May/June – 2012)**

Answer:

A Boolean algebra is a complemented, distributive lattice.

15. **Determine whether the following posets are lattices.**

(i) ({1,2,3,4,5},/) **(ii) ({1,2,4,8,16},/)**

Answer:

(i) ({1,2,3,4,5},/) is not a lattice because there is no upper bound for the pairs (2,3) and (3,5).

(ii) ({1,2,4,8,16},/) is a lattice. Since every pair has a LUB and a GLB

16. **State the De Morgan"s Law in Boolean algebra** **(Nov/Dec – 2016)**

Answer:

$(a \vee b)' = a' \wedge b'$, $(a \wedge b)' = a' \vee b'$

17. **Simplify the Boolean expression** $a'.b'.c + a.b'.c + a.b'.c'$ **, using Boolean algebraic identities.**

Answer:

$$a'.b'.c + a.b'.c + a.b'.c' = a'.b'.c + a.b'.(c + c')$$
$$= a'.b'.c + a.b'.1$$
$$= b'.(a + a'.c)$$
$$= b'.(a + a')(a.c)$$
$$= a.b' + b'.c$$

18. **Prove that any lattice homomorphism is order preserving. Answer:**

Let $f: L_1 \to L_2$ be a homomorphism.

Let $a \le b$ Then GLB }= $a \wedge b$ =a, LUB }= $a \vee b$ =b

Now $f(a \wedge b) = f(a) \Rightarrow f(a) \wedge f(b) = f(a)$

i.e., GLB { Therefore $f(a) \le f(b)$

If $a \le b$ implies $f(a) \le f(b)$. Therefore f is order preserving.

19. **Is a Boolean algebra contains six elements? Justify your answer.** **(Nov/Dec – 2015)**

Answer:

No. A Boolean algebra consists of 2^n elements.

20. **Show that the absorption laws are valid in a Boolean algebra** **(May/June – 2016)**

Answer:

Consider $a + ab = a(1 + b) = a(1) = a$

PART B

1. Prove that De Morgan's laws hold good for a complemented lattice $\langle L, \wedge, \vee \rangle$, viz $(a \vee b)' = a' \wedge b'$ and $(a \wedge b)' = a' \vee b'$.

2. If P(S) is the power set of a set S and $\cup, \cap$ are taken as join and meet, prove that

$\langle P(S), \subseteq \rangle$ is a lattice. Also, prove the modular inequality $\forall a,b,c \in L,\ a \le c \Leftrightarrow a \vee (b \wedge c) \le (a \vee b) \wedge c$.

3. In a distributive lattice $\langle L, \wedge, \vee \rangle$ if an element a$\in$ L has a complement then it is unique.
4. If S_{42} is the set of all divisors of 42 and D is the relation "divisor of "on S_{42}, prove that $\langle S_{42}, D \rangle$ is a complemented lattice.
5. Show that every chain is a distributive lattice.
6. Prove that D_{110}, the set of all divisors of 110, is a Boolean algebra and find all its sub algebras.
7. In a Boolean algebra show that $ab'+a'b=0$ if and only if a = b.
8. State and prove De Morgan's law for Boolean algebra.
9. Show that every distributive lattice is modular. Whether the converse is true? Justify your claim.
10. Prove that any chain is modular lattice.
11. Prove that the set of all +ve integers ordered by divisibility is a distributive lattice.
12. Draw the Hasse diagram of the lattice L of all subsets of {a,b,c} under intersection & Union.
13. Let $(L,*,\oplus)$ be an algebraic lattice. If we define $x \le y \Leftrightarrow x * y = x\ (or)\ x \le y \Leftrightarrow x \oplus y = y$, then prove that $(L, \le)$ is lattice ordered set.
14. In a lattice $(L, \le)$ prove that $x \vee (y \wedge z) \le (x \vee y) \wedge (x \vee z)$.
15. State and prove isotonic property for lattices.